M534H HOMEWORK– Spring 2026

Prof. Andrea R. Nahmod

• Set 1. Due date: Thursday 02/12/2026 by 5pm

Section 1.1: 2, 3, 4.

Note Note that an equivalent manner (that what we discussed in Lecture 1) of viewing linearity is explained in Strauss' book page 2.

Assigned Background Reading: Review carefully Appendices A1. A2 and A3 in Strauss' book. We will need and use this material through the course. You will also need it to understand the first three Handouts/Section 1.3 of Strauss.

Problem: Do but do not turn in: Prove the *Second Vanishing Theorem* in A.1 page 416

• Set 2. Due date: Tuesday 02/17/2026 by 5pm

Section 1.1: 11, 12.

Section 1.2: 1, 2, 3, 4, 5, 6.

Additional Problem 1: Solve the transport equation $5u_x - 6u_y = 0$ together with the auxiliary condition that $u(x, 0) = 4x^3$.

Additional Problem 2: Solve the inhomogeneous transport equation $2u_x + 3u_y = 1$.

Assigned Reading for next week: Read carefully Handouts 1, 2 and 3 posted in the webpage. These correspond to Examples 2, 3 and 4 in Strauss' Section 1.3

• Set 3. Due date: Thursday 02/26/2026 by 5pm

Section 1.2: 7

Additional Problem 1: Solve the linear homogeneous equation $u_x - u_y = 10$

Additional Problem 2: Solve the linear homogeneous equation $u_x - u_y + u = 0$.

Specify what method you are using and explain step by step your work. Show all your work.

Additional Problem 3: a) Find the general solution to

$$2u_x + 5u_y + 29u = 0$$

Specify what method you are using and explain step by step your work. Show all your work.

b) Find the solution $u(x, y)$ to part a) that also satisfies $u(x, 0) = e^{-3x}$.

Additional Problem 4: a) Check that

$$u(x, y) = \frac{1}{4}(e^{x+2y} - e^{x-2y})$$

solves the inhomogeneous equation

$$u_x + u_y + u = e^{x+2y}.$$

b) Next use first the same method used for Additional Problem 2 (but suitably change the coefficients) to then write the general form of the solutions to

$$u_x + u_y + u = e^{x+2y}.$$

c) Find the solution to $u_x + u_y + u = e^{x+2y}$ that also satisfies $u(x, 0) = 1$.

- **Set 4. Due date: Thursday 03/05/2026 by 5pm**

Section 1.3: 6, 8.

Section 1.4: 1

Section 1.5: 1, 5, and:

Problem 6 (modified): Solve the equation $u_x + 2x\,y^2\,u_y = 0$ and find a solution that satisfies the auxiliary condition $u(0, y) = y$.

- **Set 5. Due date: Thursday 03/12/2026 by 5pm**

Reminder: Show all your work justifying your answers.

Section 1.6: 1, 2, 4, 6.

Additional Problem 1: Find the regions in $\mathbb{R}^2$ where $x^2\,u_{xx} + 4u_{xy} + y^2\,u_{yy} = 0$ is respectively elliptic, parabolic, hyperbolic. Plot these regions.

Section 2.1: 1, 2, 7.

Recall: A function $f : \mathbb{R} \to \mathbb{R}$ is said to be even if $f(x) = f(-x)$; while $f : \mathbb{R} \to \mathbb{R}$ is said to be odd if $f(x) = -f(-x)$.

Additional Problem 2 Find the solution to the wave equation $u_{tt} - 4u_{xx} = 0$, $x \in \mathbb{R}$, $t \geq 0$ with initial conditions $u(x, 0) = \sin x$, $u_t(x, 0) = 10$. using D'Alambert's formula. Then calculate then $u_t(0, t)$.

Additional Problem 3 Show that if $u(x, t)$ is a solution to $u_{tt} - c^2 u_{xx} = 0$, $x, t \in \mathbb{R}$ then $u(x, -t)$ is also a solution to $u_{tt} - c^2 u_{xx} = 0$.

Note. This shows that solutions to the wave equation are *time reversible*. So once you find the solution for $t > 0$ you know what the solution should be for $t < 0$ (assuming the initial conditions are given at $t = 0$.)

- **Set 6. Due date: Tuesday 03/24/2026 by 8pm**

Section 2.1: 9, 10, 11

Hint for 11: $-\frac{1}{16} sin(x + t)$ is a particular solution. Check it!.

Additional Problem 1: Find the general solution to $u_{xx} + u_{xt} - 10u_{tt} = 0$ (check whether is hyperbolic first).

Additional Problem 2: Find the solution the IVP

$$\begin{cases} u_{xx} - 6u_{xt} + 5u_{tt} = 0, & x \in \mathbb{R},\ t > 0 \\ u(x, 0) = x^2 & \\ u_t(x, 0) = 0 & \end{cases}$$

Check first whether the second order PDE is hyperbolic.

Section 2.2: 1, 2, 3

Additional Problem 3: Let $u = u(\mathbf{x}, t)$ be a solution to the wave equation $u_{tt} - \Delta u = 0$ in $\mathbb{R}^2$. Assuming that $|\nabla u| \to 0$ fast enough as $|\mathbf{x}| \to \infty$ prove that

$$E(t) = \int_{\mathbb{R}^2} |u_t|^2 + |\nabla u|^2 \, dxdy$$

is constant in t for all time t.

Additional Problem 4 Consider the wave equation in 1D with *damping*

$$u_{tt} = c^2 u_{xx} - ku - ru_t \qquad k, r > 0$$

show that the *energy functional*

$$E(t) = \frac{1}{2}\int_{-\infty}^{\infty} |u_t|^2 + c^2|u_x|^2 + k|u|^2\, dx$$

satisfies $dE/dt \leq 0$; that is *energy decreases.* Assume u and its derivatives vanish as $x \to \pm\infty$.